Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

in n points. These points determine n "sects" measured from the fixed point and their algebraic sum represents a point on the revolving line. What is the curve which this point describes?

2. Find a geometrical construction for the following problem: Given the distances AO, BO, CO of the points A, B, C of an equilateral triangle from a fixed point O. Construct the equilateral triangle, or triangles satisfying these conditions.

3. What is the locus of the points from which any two sects in space AB and CD (not in the same plane) appear under the same constant angle?

◆

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M., (Princeton); Ph. D., (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from April Number.]

PROPOSITION XXIV. *The same hypothesis remaining; I say the four angles together (Fig. 27.) of the quadrilateral KDHK nearer the base AB are less (in hypothesis of acute angle) than the four angles together of the quadrilateral KHLK more remote from the same base; and indeed this is so, whether those two AX, BX somewhere at a finite distance meet toward the parts of the point X; or never meet one another; but toward those parts either ever more mutually approach each other, or somewhere receive a common perpendicular, after which of course (in accordance with Cor. II. of the preceding proposition) toward the same parts they begin mutually to separate.*

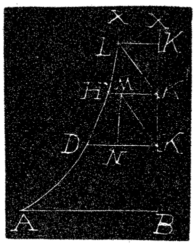

Fig. 27.

PROOF. Here however we suppose the portions KK assumed to be mutually equal. Since therefore (from the preceding) the side DK is greater than the side HK, and similarly HK greater than the side LK, the portion MK in HK is assumed equal to LK, and in DK the portion NK equal to HK; and MN, MK, LK are joined, truly the intermediate point K with the point L, and the point K near to the point B with the point M.

Now I proceed thus.

Since indeed the sides of the triangle KKL (I make beginning always from the point K nearer the point B) are equal to the sides of the triangle KKM, and the included angles equal, as being right, equal also will be (from Eu. I. 4) the bases LK, MK, and likewise equal the angles which correspond mutually, at these bases, indeed the angle *KLK* to the angle *KMK*, and the angle *LKK* to the angle *MKK*. Therefore equal also are the remainders *NKM* and *HKL*. Wherefore, since the sides *NK*, *KM* of the triangle *NKM* are equal in the same way to the sides *HK*, *KL* of the triangle *HKL*, equal also will be (from the same Eu. I. 4) the bases *NM*, *HL*, the angles *KNM*, *KHL*, and finally the angles *KMN*, *KLH*. But in the preceding triangles are already proved equal the angles *KLK*, *KMK*. Therefore the whole angle *NMK* is equal to the whole angle *HLK*.

Wherefore, since all angles at the points *K* are right, it follows manifestly all four angles together of the quadrilateral *KNMK* are equal to all four angles together of the quadrilateral *KHLK*.

But since the two angles together at the points *N* and *M* in the quadrilateral *KNMK* are greater, in hypothesis of acute angle, than the two angles together (from Cor. after P. XVI) at the points *D* and *H* in the quadrilateral *NDHM*, or the quadrilateral *KDHK*, the consequence thence is, that (the common right angles at the points *K* being added) the four angles together of the quadrilateral *KNMK*, or the quadrilateral *KHLK* are greater (in hypothesis of acute angle) than the four angles together of the quadrilateral *KDHK*.

Quod erat demonstrandum.

COROLLARY.

But it ought here opportunely to be observed, nothing will fail in the argument made, although the angle at the point *L* is assumed right, together with hypothesis of acute angle. For still that common perpendicular *LK* would be less (from Cor. I. after II of this) than the other perpendicular *HK*, from which therefore still a portion *MK* could be assumed equal to the aforesaid *LK*.

Which standing, it follows that no hindrance can intervene.

[To be Continued.]

INTRODUCTION TO SUBSTITUTION GROUPS.

By G. A. MILLER, Ph. D., Leipzig, Germany.

[Continued from April Number.]

PRIMITIVE GROUPS OF TWO, THREE, AND FOUR, LETTERS.

Since all of these must contain substitutions of the form a_1a_2 or of the form $a_1a_2a_3$ they must all contain the symmetric group. The following is therefore a complete list: